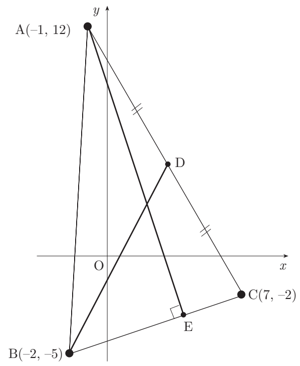
1 Triangle ABC has vertices A(–1,12), B(–2, –5) and C(7, –2).

(a) Find the equation of the median BD. **3**

(b) Find the equation of the altitude AE. **3**

(c) Find the coordinates of the point of intersection of BD and AE. **3**



Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
1	a,b,c	3,3,3	C	G7, G8	CN	06/01

The primary method m/s is based on the following generic m/s. THIS GENERIC M/S MAY BE USED AS AN EQUIVALENCE GUIDE BUT ONLY WHERE A CANDIDATE DOES NOT USE THE PRIMARY METHOD OR ANY ALTERNATIVE METHOD SHOWN IN DETAIL IN THE MARKING SCHEME

- $\bullet^1$ ic interpret "median"
- $\bullet^2$ ss find gradient
- $\bullet^3$ ic state equation
- $\bullet^4$ ss find gradient
- $\bullet^5$ ss find perpendicular gradient
- $\bullet^6$ ic state equation
- $\bullet^7$ ss start to solve simultaneous equations
- $\bullet^8$ pr solve for one variable
- $\bullet^9$ pr process

Primary Method : Give 1 mark for each ·

- $\bullet^1$ $D=(3,5)$
- $\bullet^2$ $m_{BD}=2$
- $\bullet^3$ $y-5=2(x-3)$ *or* $y+5=2\big(x-(-2)\big)$ *etc* **3 marks**
- $\bullet^4$ $m_{BC}=\frac{1}{3}$ **stated explicitly**
- $\bullet^5$ $m_{alt}=-3$
- $\bullet^6$ $y-12=-3(x-(-1))$ **3 marks**
- $\bullet^7$ $y-5=2(x-3)$ ***and*** $y-12=-3(x-(-1))$ **or equivalent**
- $\bullet^8$ $x=2$
- $\bullet^9$ $y=3$ **3 marks**

Notes

1 For candidates who find two medians $\bullet^1,\bullet^2,\bullet^3$ and $\bullet^7,\bullet^8,\bullet^9$ are available.

2 For candidates who find two altitudes $\bullet^4,\bullet^5,\bullet^6$ and $\bullet^7,\bullet^8,\bullet^9$ are available.

3 For candidates who find (a) altitude and (b) median see common error box number 3.

4 In (a) note that (4, 7) happens to lie on the median but does not qualify as a point to be used in $\bullet^3$.

Notes cont

5 In (b) $\bullet^6$ is only available as a consequence of attempting to find a perpendicular gradient.

6 In (b) candidates who guess the coordinates for E and use these to find the equation AE, can earn no marks in this part.

7 In (c) note that "equating zeros" is only a valid strategy when either the coefficients of x or the coefficients of y are equal.

8 $\bullet^7$ is a strategy mark for juxtaposing the two required equations.

9 See general note at the foot of page 7.

Common Error 1
Finding two medians

- $\bullet^1$ $D=(3,5)$
- $\bullet^2$ $m_{BD}=2$
- $\bullet^3$ $y-5=2(x-3)$
- $\bullet^4$ X
- $\bullet^5$ X
- $\bullet^6$ X
- $\bullet^7$ $y=2x-1$ & $31x+7y=53$
- $\bullet^8$ $x=\dfrac{4}{3}$
- $\bullet^9$ $y=\dfrac{5}{3}$

maximum of 6 marks

Common Error 2
Finding two altitudes

- $\bullet^1$ X
- $\bullet^2$ X
- $\bullet^3$ X
- $\bullet^4$ $m_{BC}=\frac{1}{3}$
- $\bullet^5$ $m_{alt}=-3$
- $\bullet^6$ $y-12=-3(x-(-1))$
- $\bullet^7$ $4x-7y=27$ & $y=-3x+9$
- $\bullet^8$ $x=\dfrac{18}{5}$
- $\bullet^9$ $y=-\dfrac{9}{5}$

maximum of 6 marks

Common Error 3
Finding (a) altitude and (b) median

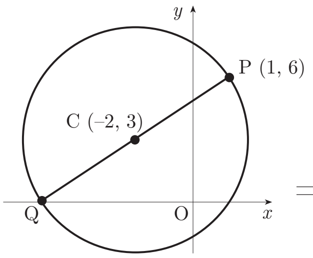
	$\bullet^1$	$m_{AC}=-\frac{7}{4}$
$X\checkmark$	$\bullet^2$	$m_{BD}=\frac{4}{7}$
	$\bullet^3$	$y--5=\frac{4}{7}(x--2)$
$X\checkmark$	$\bullet^4$	$midpt\ of\ BC=\left(\frac{5}{2},-\frac{7}{2}\right)$
	$\bullet^5$	$m_{AC}=-\frac{31}{7}$
	$\bullet^6$	$y-12=-\frac{31}{7}(x-(-1))$
$X\checkmark$	$\bullet^7$	$4x-7y=27$ & $31x+7y=53$
$X\checkmark$	$\bullet^8$	$x=\frac{16}{7}$
$X\checkmark$	$\bullet^9$	$y=-\frac{125}{49}$

maximum of 5 marks

2 A circle has centre C(–2, 3) and passes through P(1, 6).

(a) Find the equation of the circle. **2**

(b) PQ is a diameter of the circle. Find the equation of the tangent to this circle at Q. **4**



Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
2	a	2	C	G10	CN	06/54
	b	4	C	G11	CN	

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- •1 ic enter coord. of centre in general equation
- •2 ss find (radius)²
- •3 ss e.g. use $\overrightarrow{PC} = \overrightarrow{CQ}$ to find Q
- •4 pr find gradient of diameter
- •5 ss know and use tangent perp. to diameter
- •6 ic state equation

Primary Method : Give 1 mark for each •

$(x-a)^2 + (y-b)^2 = r^2$

•1 $(x-(-2))^2 + (y-3)^2$

•2 $r^2 = 18$

2 marks

•3 $Q = (-5, 0)$

•4 $m_{\text{diameter}} = 1$ stated or implied by •5

•5 $m_{\text{tangent}} = -1$

•6 $y - 0 = -\left(x-(-5)\right)$

4 marks

Notes

1 In (a) $(\sqrt{18})^2$ is not acceptable for •2.

2 In (b) if the coordinates of Q are estimated (i.e. guessed) then •6 can only be awarded if the coordinates are of the form (a, 0) where a < –2.

3 In (b) •6 is only available if an attempt has been made to find a perpendicular gradient.

Alternative Method for (a)

For answers of the form $x^2 + y^2 + 2gx + 2fy + c = 0$

•1 $x^2 + y^2 + 4x - 6y + c = 0$

•2 $c = -5$

General Notes applicable throughout the marking scheme

There are many instances when follow throughs come into play and these will not always be highlighted for you. The following example is a reminder of what you have to look out for when you are marking.

example
At the •3 stage a candidate start with the wrong coordinates for Q.
Then

X •3 $Q = (-4, 0)$

$X\checkmark$ •4 $m_{\text{diameter}} = \frac{6}{5}$

$X\checkmark$ •5 $m_{\text{tangent}} = -\frac{5}{6}$

$X\checkmark$ •6 $y - 0 = -\frac{5}{6}\left(x-(-4)\right)$

so the candidate loses •3 but gains •4, •5 and •6 as a consequence of following through.
Any error can be followed through and the subsequent marks awarded provided the working has not been eased.
Any deviation from this will be noted in the marking scheme.

3 Two functions f and g are defined on the set of real numbers by
$f(x) = 2x + 3$ and $g(x) = 2x - 3$.

(a) Find an expressions for (i) $f\big(g(x)\big)$ (ii) $g\big(f(x)\big)$. **3**

(b) Determine the least possible value of $f\big(g(x)\big) \times g\big(f(x)\big)$. **2**

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
3	a	3	C	A4	CN	06/07
	b	2	C	A6	CN	

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- $\bullet^1$ ic int. composition
- $\bullet^2$ ic int. composition
- $\bullet^3$ ic int. composition
- $\bullet^4$ pr simplify all functions
- $\bullet^5$ ic int. result

Primary Method : Give 1 mark for each •

$\bullet^1$ $f\big(g(x)\big) = f(2x-3)$ **stated or implied by •2**

$\bullet^2$ $2(2x-3)+3$

$\bullet^3$ $g\big(f(x)\big) = 2(2x+3)-3$ **3 marks**

$\bullet^4$ $16x^2 - 9$ **stated explicitly**

$\bullet^5$ $\text{min.value} = -9$ **2 marks**

Notes

1 In (a) 2 marks are available for finding one of $f\big(g(x)\big)$ *or* $g\big(f(x)\big)$ and the third mark is for the other one.

2 In (a) the finding of $f\big(f(x)\big)$ and $g\big(g(x)\big)$ earns no marks.

3 •5 is only available if •4 has been awarded.

4 In (b) for •5, no justification is necessary.
Ignore any comments, rational or irrational.

Alternative Marking 1 [Marks 1-3]

$\bullet^1$ $g\big(f(x)\big) = g(2x+3)$

$\bullet^2$ $2(2x+3)-3$

$\bullet^3$ $f\big(g(x)\big) = 2\big(2x-3\big)+3$

Common Error No.1 for (a) "g and f" transposed.

X	$\bullet^1$	$f\big(g(x)\big) = f(2x+3)$
$\surd\,X$	$\bullet^2$	$2(2x+3)-3$
$\surd\,X$	$\bullet^3$	$g\big(f(x)\big) = 2(2x-3)+3$

Award 2 out of 3

Common Error No.2 for (a)

X	$\bullet^1$	$f\big(g(x)\big) = f(2x+3)$
$\surd\,X$	$\bullet^2$	$2(2x+3)-3$
$\surd$	$\bullet^3$	$g\big(f(x)\big) = 2(2x+3)-3$

Award 2 out of 3

Common Error No.3 for (a) Repeated error

$\surd$	$\bullet^1$	$f\big(g(x)\big) = f(2x-3)$
X	$\bullet^2$	$2(2x+3)-3$
$\surd\,X$	$\bullet^3$	$g\big(f(x)\big) = 2(2x-3)+3$

Award 2 out of 3

4 A sequence is defined by the recurrence relation $u_{n+1} = 0.8u_n + 12,\ u_0 = 4$.

(a) State why the recurrence relation has a limit. **1**

(b) Find this limit. **2**

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
4	a	1	C	A12	NC	06/28
	b	2	C	A13	NC	

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- $\bullet^1$ ic state limit condition
- $\bullet^2$ ss know how to find L
- $\bullet^3$ pr process limt

Primary Method : Give 1 mark for each •

- $\bullet^1$ sequence has limit since $-1 < 0.8 < 1$ — **1 mark**
- $\bullet^2$ $L = 0.8L + 12$
- $\bullet^3$ limit $= 60$ — **2 marks**

Notes

For (a)

1 **Accept**

$|0.8| < 1$

$0 < 0.8 < 1$

0.8 lies between -1 and 1

0.8 is a proper fraction

2 **Do NOT accept**

$-1 \le 0.8 \le 1$

$-1 < a < 1$ unless a is clearly identifed/replaced by 0.8 anywhere in the answer.

$0.8 < 1$

In (b)

3 $L = \dfrac{b}{1-a}$ and nothing else gains **no** marks.

4 $L = \dfrac{12}{0.2}$ *or* $\dfrac{120}{2}$ *or* $\dfrac{60}{1}$ etc does **NOT** gain $\bullet^3$.

5 An answer of 60 without any working gains **NO** marks.

6 Any calculations based on "wrong" formulae gain **NO** marks.

Alternative Method for (b)

- $\bullet^2$ $L = \dfrac{12}{1-0.8}$
- $\bullet^3$ $limit = 60$

Bad Form

- $\bullet^2$ $L = \dfrac{12}{0.2}$
- $\bullet^3$ $limit = 60$

award 2 marks

Common Error 1

- X $\bullet^2$ $L = \dfrac{4}{1-0.8}$
- $X\checkmark$ $\bullet^3$ $limit = 20$

5 A function f is defined by $f(x) = \left(2x-1\right)^5$. Find the coordinates of the stationary point on the graph with equation $y = f(x)$ and determine its nature. **7**

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
5		6	C	C8, C9	NC	06/76
		1	B			

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- $\bullet^1$ ss know to start to differentiate
- $\bullet^2$ pr differentiate
- $\bullet^3$ ss set derivative = 0
- $\bullet^4$ pr solve
- $\bullet^5$ pr evaluate
- $\bullet^6$ ic justification
- $\bullet^7$ ic state conclusion

Primary Method : Give 1 mark for each •

- $\bullet^1$ $f'(x) = ...$
- $\bullet^2$ $5(2x-1)^4 \times 2$
- $\bullet^3$ $f'(x) = 0$
- $\bullet^4$ $x = \frac{1}{2}$
- $\bullet^5$ $f(\frac{1}{2}) = 0$
- $\bullet^6$ nature table
- $\bullet^7$ pt of inflexion at $(\frac{1}{2},0)$ **7 marks**

Notes

1 The "= 0" shown at $\bullet^3$ must appear at least once somewhere in the working between $\bullet^1$ and $\bullet^4$ (but not necessarily at $\bullet^3$).

2 $\bullet^4$ is only available as a consequence of solving $f'(x) = 0$.

3 A wrong derivative which eases the working will preclude at least $\bullet^4$ from being awarded.

4 For marks $\bullet^6$ and $\bullet^7$, a nature table is mandatory. The minimum amount of detail that is required is shown here:

	$<\frac{1}{2}$	$\frac{1}{2}$	$>\frac{1}{2}$
$f'(x)$	$+$	0	$+$
	$\iddots$	$\cdots$	$\iddots$

Candidates who use only $f''(x) = 0$ and try to draw conclusions from this cannot gain $\bullet^6$ or $\bullet^7$.
[$f''(x) = 0$ is a necessary but not sufficient condition for identifying points of inflexion].

5 $\bullet^7$ is **ONLY** available subsequent to a correct nature table for the candidate's own derivative.

6 $\bullet^4$ is lost in each of the following cases for the candidate's solution to the equation at $\bullet^3$.

(i) $x = \frac{1}{2}$ *and* $x =$ *something else*
(ii) *two wrong values for* x
(iii) *guess a value for* x

Only one value for x needs to be followed through for $\bullet^5$, $\bullet^6$ and $\bullet^7$.

Common Error No.1

√	$\bullet^1$	$f'(x) = ...$
X	$\bullet^2$	$5(2x-1)^4$
√	$\bullet^3$	$f'(x) = 0$
X √	$\bullet^4$	$x = \frac{1}{2}$

$\bullet^5$, $\bullet^6$ *and* $\bullet^7$ *are still available*

Common Error No.2

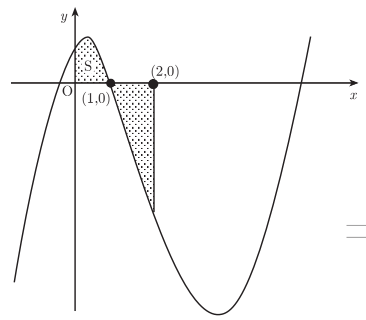
√	$\bullet^1$	$f'(x) = ...$
X	$\bullet^2$	$\frac{1}{12}(2x-1)^6$
√	$\bullet^3$	$f'(x) = 0$
X √	$\bullet^4$	$x = \frac{1}{2}$

$\bullet^5$, $\bullet^6$ *and* $\bullet^7$ *are still available*

6 The graph shown has equation $y = x^3 - 6x^2 + 4x + 1$.
The shaded area is bounded by the curve, the x-axis, the y-axis and the line $x = 2$.

(a) Calculate the shaded area labelled S. **4**

(b) Hence find the total shaded area. **3**



Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
6	a	4	C	C16	NC	06/40
	b	3	B	C16	NC	

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- $\bullet^1$ ss know to integrate
- $\bullet^2$ pr integrate
- $\bullet^3$ ic substitute limits
- $\bullet^4$ pr evaluate
- $\bullet^5$ ic use result from $\bullet^2$ with new limits
- $\bullet^6$ pr evaluate
- $\bullet^7$ ss deal with the "–ve" sign and evaluate total area

Primary Method : Give 1 mark for each •

$\bullet^1$ $\int_0^1 \left(x^3 - 6x^2 + 4x + 1\right)dx$ stated or implied by $\bullet^2$

$\bullet^2$ $\frac{1}{4}x^4 - \frac{6}{3}x^3 + \frac{4}{2}x^2 + x$

$\bullet^3$ $\left(\frac{1}{4}.1^4 - 2.1^3 + 2.1^2 + 1\right) - 0$

$\bullet^4$ $\frac{5}{4}$ **or equivalent** **4**

$\bullet^5$ $\int_1^2 ...dx$

$\bullet^6$ $\left(\frac{1}{4}.2^4 - 2.2^3 + 2.2^2 + 2\right) - \left(\frac{1}{4}.1^4 - 2.1^3 + 2.1^2 + 1\right) = -\frac{13}{4}$

$\bullet^7$ $\frac{9}{2}$ **or equivalent** **3**

Notes for (a)

1 Only a limited number of marks are available to candidates who differentiate –see Common Error No.1.

2 In (a)
candidates who transpose the limits can still earn $\bullet^4$ if the deal with the "–ve" sign appropriately.

3 In (b)
$\bullet^7$ is lost for such statements as $-3\frac{1}{4} = 3\frac{1}{4}$.

4 In (b) using $\int_0^2 ...dx$ earns no marks.

Common Error No.1

√	$\bullet^1$	$\int_0^1 \left(x^3 - 6x^2 + 4x + 1\right)dx$
X	$\bullet^2$	$3x^2 - 12x + 4$
X	$\bullet^3$	$\left(3.1^2 - 12.1 + 4\right) - 4$
X	$\bullet^4$	-9
√	$\bullet^5$	$\int_1^2 ...dx$ *or equivalent*
X √	$\bullet^6$	$\left(3.2^2 - 12.2 + 4\right) - \left(3.1^2 - 12.1 + 4\right) = -3$
X √	$\bullet^7$	12

Alternative Method 1 for (b)

$\bullet^5$ $\int_2^1 ...dx$

$\bullet^6$ $\left(\frac{1}{4}.1^4 - 2.1^3 + 2.1^2 + 1\right) - \left(\frac{1}{4}.2^4 - 2.2^3 + 2.2^2 + 2\right)$

$\bullet^7$ $\frac{9}{2}$

Alternative Method 2 for (b)

$\bullet^5$ $-\int_1^2 ...dx$

$\bullet^6$ $-\left(\frac{1}{4}.2^4 - 2.2^3 + 2.2^2 + 2\right) + \left(\frac{1}{4}.1^4 - 2.1^3 + 2.1^2 + 1\right)$

$\bullet^7$ $\frac{9}{2}$

Alternative Method 3 for (b)

$\bullet^5$ $\left|\int_1^2 ...dx\right|$

$\bullet^6$ $\left|\left(\frac{1}{4}.2^4 - 2.2^3 + 2.2^2 + 2\right) - \left(\frac{1}{4}.1^4 - 2.1^3 + 2.1^2 + 1\right)\right|$

$\bullet^7$ $\frac{9}{2}$

7	Solve the equation $\sin x° - \sin 2x° = 0$ in the interval $0 \le x \le 360$.	4

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
7		4	C	T10	NC	06/46

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- $\bullet^1$ ss know to use double angle formula
- $\bullet^2$ pr factorise
- $\bullet^3$ pr solve
- $\bullet^4$ ic know exact values

Primary Method : Give 1 mark for each •

$\bullet^1$ $\sin(x°) - 2\sin(x°)\cos(x°) = 0$

$\bullet^2$ $\sin(x°)\big(1 - 2\cos(x°)\big) = 0$

$\bullet^3$ $\sin(x°) = 0 \textit{ or } \cos(x°) = 0.5$

$\bullet^4$ $x = 0, 180, 360, \quad 60, 300$ 4

Alternative Marking Method (Cross marking for •3 and •4)

$\bullet^1$ $\sin(x°) - 2\sin(x°)\cos(x°) = 0$

$\bullet^2$ $\sin(x°)\big(1 - 2\cos(x°)\big) = 0$

$\bullet^3$ $\sin(x°) = 0 \textit{ and } x = 0, 180, 360$

$\bullet^4$ $\cos(x°) = 0.5 \textit{ and } x = 60, 300$

Alternative Method Division by sin(x)

$\bullet^1$ $\sin(x°) - 2\sin(x°)\cos(x°) = 0$

$\bullet^2$ $\textit{either } \sin(x°) = 0 \textit{ or } \sin(x°) \ne 0$

$\bullet^3$ $\sin(x°) = 0 \Rightarrow x = 0, 180, 360$

$\bullet^4$ $\cos(x°) = 0.5 \Rightarrow x = 60, 300$

Notes

1. An "= 0" must appear somewhere between the start and $\bullet^2$ evidence.
2. The inclusion of extra answers which would have been correct with a larger interval should be treated as bad form and NOT penalised.
3. The omission of a correct answer (e.g. 0) means the candidates loses a mark ($\bullet^4$ in the Primary Method).
4. Candidates may embark on a journey with the wrong formula for sin(2x°). With an equivalent level of difficulty it may still be worth a maximum of 3 marks. See Common Error No.1.
5. Candidates who draw a sketch of $y = \sin(x°)$ and $y = \sin(2x°)$ giving 0,180,360 may be awarded $\bullet^1$ and $\bullet^3$.

Common Error No.1

X $\bullet^1$ $\sin(x°) - \big(1 - 2\sin^2(x°)\big) = 0$

$2\sin^2(x°) + \sin(x°) - 1 = 0$

$X\checkmark$ $\bullet^2$ $\big(2\sin(x°) - 1\big)\big(\sin(x°) + 1\big) = 0$

$X\checkmark$ $\bullet^3$ $\sin(x°) = \frac{1}{2} \textit{ or } \sin(x°) = -1$

$X\checkmark$ $\bullet^4$ $x = 30, 150, \quad x = 270$

award 3 marks

Common Error No.2

$\sin(x°) - \sin^2(x°) = 0$

X $\bullet^1$

$X\checkmark$ $\bullet^2$ $\sin(x°)\big(1 - \sin(x°)\big) = 0$

X $\bullet^3$ $\sin(x°) = 0 \textit{ or } \sin(x°) = 1$

$X\checkmark$ $\bullet^4$ $x = 0, 180, 360, \quad 90$

award 2 marks

Common Error No.3

$\sin(x) - \sin(2x) = 0$

$\sin(x) = 0, \ \sin(2x) = 0$

etc

gains NO marks

8 (a) Express $2x^2 + 4x - 3$ in the form $a(x+b)^2 + c$. **3**

(b) Write down the coordinates of the turning point on the parabola with equation $y = 2x^2 + 4x - 3$. **1**

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
8	a	3	B	A5	NC	06/32
	b	1	C	A6	NC	

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- $\bullet^1$ ss know how to complete (deal with the "a")
- $\bullet^2$ pr process the value of "b"
- $\bullet^3$ pr process the value of "c"
- $\bullet^4$ ic interpret equation of parabola

Primary Method : Give 1 mark for each •

- $\bullet^1$ $a = 2$
- $\bullet^2$ $b = 1$
- $\bullet^3$ $c = -5$ **3**
- $\bullet^4$ $(-1, -5)$ **1**

Alternative Method 1 for (a)

- $\bullet^1$ $2(x^2 + 2x)$
- $\bullet^2$ $2(x+1)^2$
- $\bullet^3$ $2(x+1)^2 - 5$
- $\bullet^4$ $(-1, -5)$

Note

1 Alternative Method 1 should be used for assessing part marks/follow throughs.

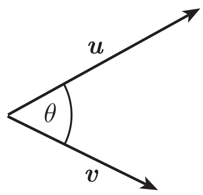
2 For $\bullet^4$, no justification is required.
Candidates may choose to differentiate etc. but may still earn only one mark for the correct answer.

3 For $\bullet^4$, accept (–b, c).

Alternative Method 2 for (a) : Comparing coefficients

- $\bullet^1$ $2x^2 + 4x - 3 = ax^2 + 2abx + ab^2 + c \quad \Rightarrow a = 2$
- $\bullet^2$ $2ab = 4 \quad \Rightarrow b = 1$
- $\bullet^3$ $ab^2 + c = -3 \quad \Rightarrow c = -5$
- $\bullet^4$ $(-1, -5)$

9 $\boldsymbol{u}$ and $\boldsymbol{v}$ are vectors given by $\boldsymbol{u} = \begin{pmatrix} k^3 \\ 1 \\ k+2 \end{pmatrix}$ and $\boldsymbol{v} = \begin{pmatrix} 1 \\ 3k^2 \\ -1 \end{pmatrix}$, where $k > 0$.



(a) If $\boldsymbol{u}.\boldsymbol{v} = 1$ show that $k^3 + 3k^2 - k - 3 = 0$. **2 marks**

(b) Show that $(k+3)$ is a factor of $k^3 + 3k^2 - k - 3$ and hence factorise $k^3 + 3k^2 - k - 3$ fully. **5 marks**

(c) Deduce the only possible value of k. **1 mark**

(d) The angle between $\boldsymbol{u}$ and $\boldsymbol{v}$ is θ. Find the exact value of $\cos\theta$. **3 marks**

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
8	a	2	C	G26	CN	05/10
	b	5	C	A21	NC	
	c	1	C	A6	CN	
	d	3	C	G28	NC	

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- •[1] pr find scalar product
- •[2] ic complete proof
- •[3] ss know to use $k = -3$
- •[4] pr complete evaluation and conclusion
- •[5] ic start to find quadratic factor
- •[6] ic complete quadratic factor
- •[7] pr factorise completely
- •[8] ic interpret k
- •[9] ic interpret vectors
- •[10] pr find magnitudes
- •[11] ss use formula

Primary Method : Give 1 mark for each •

- •[1] $\boldsymbol{u}.\boldsymbol{v} = k^3.1 + 1.\left(3k^2\right) + \left(k+2\right).(-1)$ stated or implied by •2 before completion
- •[2] $k^3 + 3k^2 - k - 2 = 1$ *and complete* **2 marks**

- •[3] know to use $k = -3$
- •[4] $-27 + 27 - (-3) - 3 = 0 \Rightarrow x + 3$ *is a factor*
- •[5] $(k+3)(k^2 \;....)$
- •[6] $(k+3)\left(k^2 - 1\right)$
- •[7] $(k+3)(k+1)(k-1)$ **stated explicitly** **5 marks**

- •[8] $k = 1$ **1 mark**

- •[9] $\boldsymbol{u} = \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix}, \boldsymbol{v} = \begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix}$ **stated or implied by •10**
- •[10] $|\,\boldsymbol{u}\,| = \sqrt{11}$ *and* $|\,\boldsymbol{v}\,| = \sqrt{11}$
- •[11] $\cos\theta = \dfrac{1}{11}$ **3 marks**

N.B.
•9 and •10 may be cross-marked.

Notes

1 No explanation is required for k but the chosen value must follow from the working for •6 or •7. **Do not accept √1.**

2 In primary method (•4) and alternative (•5) candidates must show some acknowledgement of the resulting "zero". Although a statement w.r.t. the zero is preferable, accept something as simple as "underlining" the zero.

3 Only numerical values are acceptable for •9, •10 and •11; answers are acceptable in unsimplified form eg $\cos\theta = \frac{1}{\sqrt{11}\times\sqrt{11}}$

Alternative method 1 (marks 3–7) Long Division

•[3]
$$\begin{array}{r|llll} & k^2 & & & -1 \\ \hline k+3 & k^3 & +3k^2 & -k & -3 \\ & k^3 & +3k^2 & & \\ & ..... & .... & .... -k & -3 \end{array}$$

•[4]
$$\begin{array}{ll} -k & -3 \end{array}$$

- •[5] *remainder is zero so* $(k+3)$ *is a factor*
- •[6] $k^2 - 1$
- •[7] $(k+3)(k+1)(k-1)$ **stated explicitly**

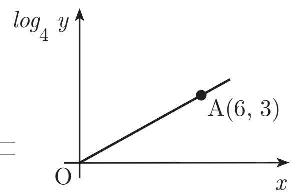
Alternative method 2 (marks 3–7) Synthetic Division

•[3]
$$\begin{array}{r|l} -3 & \\ \hline & \end{array}$$

•[4]
$$\begin{array}{r|rrrr} -3 & 1 & 3 & -1 & -3 \\ & & -3 & 0 & 3 \\ \hline & 1 & 0 & -1 & 0 \end{array}$$

- •[5] $"f(-3)" = 0$ *so* $(k+3)$ *is a factor*
- •[6] $\left(k^2 - 1\right)$
- •[7] $(k+3)(k+1)(k-1)$ **stated explicitly**

10 Two variables, x and y, are connected by the law $y = a^x$. A graph of $\log_4(y)$ against x is a straight line passing through the origin and the point A(6,3). Find the value of a. **4**



Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
10		4	A	A33	NC	06/91

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- $\bullet^1$ ss know to take logarithms
- $\bullet^2$ ic substitute known point
- $\bullet^3$ pr solve
- $\bullet^4$ pr solve

Primary Method : Give 1 mark for each •

- $\bullet^1$ $\log_4(y) = \log_4(a^x)$
- $\bullet^2$ $3 = \log_4(a^6)$
- $\bullet^3$ $a^6 = 4^3$
- $\bullet^4$ $a = 2$ **4 marks**

Note

1 $m = \frac{1}{2}$ and nothing else gains no marks.

2 For $\bullet^4$, a correct answer without any legitimate evidence gains **NO** marks.

3 For $\bullet^4$, ignore the inclusion of a negative answer.

Alternative Method 1

- $\bullet^1$ $\log_4(y) = \log_4(a^x)$
- $\bullet^2$ $3 = 6\log_4(a)$
- $\bullet^3$ $\log_4(a) = \frac{1}{2}$
- $\bullet^4$ $a = 2$

Alternative Method 2

- $\bullet^1$ $\log_4(y) = mx + c$
- $\bullet^2$ $m = \frac{1}{2}$, $c = 0$
- $\bullet^3$ $y = 4^{\frac{1}{2}x}$
- $\bullet^4$ $y = \left(4^{\frac{1}{2}}\right)^x = 2^x \Rightarrow a = 2$

Common Error 1

$\surd$	$\bullet^1$	$\log_4(y) = \log_4(a^x)$
X	$\bullet^2$	$\log_4(3) = \log_4(a^6)$
X	$\bullet^3$	$3 = a^6$
X	$\bullet^4$	$a = 3^{\frac{1}{6}}$

Alternative Method 3

- $\bullet^1$ At A $\log_4(y) = 3$
- $\bullet^2$ $y = 4^3$
- $\bullet^3$ $a^6 = 4^3$
- $\bullet^4$ $a = 2$

Common Error 2

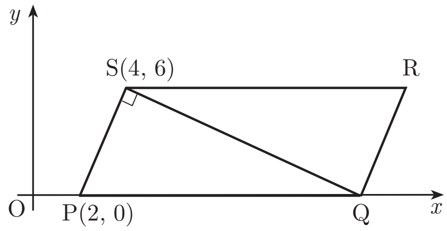
X	$\bullet^1$	$\log_4(y) = x$
X	$\bullet^2$	$--$
$X\surd$	$\bullet^3$	$y = 4^x$
X	$\bullet^4$	$a = 4$

Alternative Method 4

- $\bullet^1$ $\log_4(y) = \log_4(a^x)$
- $\bullet^2$ $\log_4(y) = x\log_4(a)$
- $\bullet^3$ $\log_4(a) = \frac{1}{2}$
- $\bullet^4$ $a = 4^{\frac{1}{2}} = 2$

1 PQRS is a parallelogram. P is the point (2, 0), S is (4, 6) and Q lies on the x-axis, as shown.

The diagonal QS is perpendicular to the side PS.



(a) Show that the equation of QS is $x + 3y = 22$. **4**

(b) Hence find the coordinates of Q and R. **2**

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
1	a,b	4,2	C	G8	CN	06/05

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- •¹ pr find gradient from two points
- •² ss use $m_1 m_2 = -1$
- •³ ic state equation of the line
- •⁴ ic completes proof
- •⁵ ic interpret diagram
- •⁶ ic interpret diagram

Primary Method : Give 1 mark for each •

- •¹ $m_{PS} = 3$
- •² $m_{QS} = -\frac{1}{3}$
- •³ $y - 6 = -\frac{1}{3}(x - 4)$
- •⁴ completes proof **4 marks**
- •⁵ $Q = (22, 0)$
- •⁶ $R = (24, 6)$ **2 marks**

Notes

In (a)

1 In the Primary method, •³ is only available if an attempt has been made to find and use a perpendicular gradient.

2 In the Primary method and the Alt. method 1, •⁴ is only available for reaching the required equation.

3 To gain •⁴, some evidence of completion needs to be shown

e.g. $y - 6 = -\frac{1}{3}(x - 4)$

$3(y - 6) = -(x - 4)$

$x + 3y = 22$

4 Sometimes candidates manage to find R first. Provided the coordinates of R are of the form (?, 6), only then is •⁶ available as a follow through.

5 •⁵ and •⁶ are available to candidates who use their own erroneous equation for QS.

General Notes applicable throughout the marking scheme

There are many instances when follow throughs come into play and these will not always be highlighted for you. The following example is a reminder of what you have to look out for when you are marking.

example

At the •⁵ stage a candidate may switch the coordinates round so we have

•⁵ X $Q(0, 22)$

•⁶ $X\checkmark$ $R(2, 28)$ *repeated error*

so the candidate loses •⁵ for switching the coordinates but gains •⁶ as a consequence of following through.

Any error can be followed through and the subsequent marks awarded provided the working has not been eased.

Any deviation from this will be noted in the marking scheme.

Alternative Method 1

- •¹ $m_{PS} = 3$
- •² $m_{QS} = -\frac{1}{3}$

  $y = -\frac{1}{3}x + c$
- •³ $6 = -\frac{1}{3} \times 4 + c$
- •⁴ completes proof
- •⁵ $Q = (22, 0)$
- •⁶ $R = (24, 6)$

Alternative Method 2

Let $Q = (q, 0)$

- •¹ $(q - 2)^2 = 2^2 + 6^2 + (q - 4)^2 + 6^2$
- •² $q = 22$
- •³ $Q = (22, 0)$ *and* $R = (24, 6)$
- •⁴ $m_{QS} = -\frac{1}{3}$
- •⁵ $y - 0 = -\frac{1}{3}(x - 22)$
- •⁶ *leading to* $3y + x = 22$

N.B.

The coordinates of Q can also be arrived at by right-angled trig.

Use the alt. method 2 marking scheme with •¹ replaced by appropriate trig. work.

The only acceptable value for q is 22.

2 Find the value of k such that the equation $kx^2 + kx + 6 = 0$, $k \neq 0$, has equal roots. **4**

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
2		4	C	A18	CN	06/new

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- $\bullet^1$ ss know to use "discriminant = 0"
- $\bullet^2$ ic interpret a, b, c
- $\bullet^3$ pr substitute & factorise
- $\bullet^4$ ic interpret solution

Primary Method : Give 1 mark for each •

$\bullet^1$ $"b^2 - 4ac" = 0$

$\bullet^2$ $a = k, b = k, c = 6$

$\bullet^3$ $k(k - 24)$

$\bullet^4$ $\left[\begin{array}{l} k = 0 \quad and \quad k = 24 \\ \quad \therefore \quad k = 24 \end{array} \right.$

4 marks

Alternative Method 1 (completing the square)

$\bullet^1$ $\left(x + \frac{1}{2}\right)^2 + \ldots\ldots$

$\bullet^2$ $\left(x + \frac{1}{2}\right)^2 - \frac{1}{4} + \frac{6}{k} = 0$

$\bullet^3$ $equal\ roots \Rightarrow -\frac{1}{4} + \frac{6}{k} = 0$

$\bullet^4$ $k = 24$

Notes

1 The evidence for •[1] and/or •[2] may not appear until the working immediately preceding the evidence for •[3].
i.e. a candidate may simply start

$\surd \bullet^1, \surd \bullet^2$ $\quad k^2 - 4 \times k \times 6 = 0$

$\surd \bullet^3$ $\quad k(k - 24)$

or

$\surd \bullet^2$ $\quad k^2 - 4 \times k \times 6$

$\surd \bullet^1, \surd \bullet^3$ $\quad k(k - 24) = 0$

2 The "= 0" has to appear at least once, at the •[1] stage or at the •[3] stage.

3 In the Primary method, candidates who do not deal with the root k = 0 cannot obtain •[4].
[see Common Errors 1 and 2]
Minimum evidence for •[4] would be scoring out "k = 0" or "k = 24" underlined.

4 Some candidates may start with the quadratic formula. Apply the marking scheme to the part underneath the square root sign.

5 The use of any expression masquerading as the discriminant can only gain •[2] at most.

Acceptable alternative for •4

$\surd$	$\bullet^1$	$"b^2 - 4ac" = 0$
$\surd$	$\bullet^2$	$a = k, b = k, c = 6$
$\surd$	$\bullet^3$	$k(k - 24)$
$\surd$	$\bullet^4$	$k \cancel{= 0}$ $or\ 24$

Common Error 1 at the •4 stage

$\surd$	$\bullet^1$	$"b^2 - 4ac" = 0$
$\surd$	$\bullet^2$	$a = k, b = k, c = 6$
$\surd$	$\bullet^3$	$k(k - 24)$
X	$\bullet^4$	$k = 0\ or\ 24$

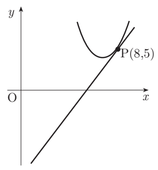
Common Error 2 at the •4 stage

$\surd$	$\bullet^1$	$"b^2 - 4ac" = 0$
$\surd$	$\bullet^2$	$a = k, b = k, c = 6$
$\surd$	$\bullet^3$	$k(k - 24)$
X	$\bullet^4$	$k = 24$

Common Error 3 Division by k

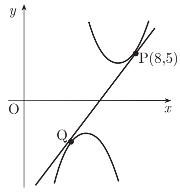
$\surd$	$\bullet^1$	$"b^2 - 4ac" = 0$
$\surd$	$\bullet^2$	$a = k, b = k, c = 6$
X	$\bullet^3$	$k^2 - 24k = 0$
		$k^2 = 24k$
X	$\bullet^4$	$k = 24$

3 The parabola with equation $y = x^2 - 14x + 53$ has a tangent at the point P(8,5).



(*a*) Find the equation of this tangent. **4**

(*b*) Show that the tangent found in (*a*) is also a tangent to the parabola with equation $y = -x^2 + 10x - 27$ and find the coordinates of the point of contact Q. **5**



Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
3	a	4	C	C5	CN	06/26
	b	5	C	A24	CN	

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- $\bullet^1$ ss know to differentiate
- $\bullet^2$ pr differentiate
- $\bullet^3$ pr evaluate gradient
- $\bullet^4$ ic state equation of tangent
- $\bullet^5$ ss arrange in standard form
- $\bullet^6$ ss substitute into quadratic
- $\bullet^7$ pr process
- $\bullet^8$ ic factorise & interpret
- $\bullet^9$ ic state coordinates

Primary Method : Give 1 mark for each •

- $\bullet^1$ $\frac{dy}{dx} =$
- $\bullet^2$ $2x - 14$
- $\bullet^3$ $m = 2$ **stated or implied by •4**
- $\bullet^4$ $y - 5 = 2(x - 8)$ **4 marks**

- $\bullet^5$ $y = 2x - 11$
- $\bullet^6$ $2x - 11 = -x^2 + 10x - 27$
- $\bullet^7$ $x^2 - 8x + 16 = 0$
- $\bullet^8$ $(x-4)^2 = 0 \Rightarrow$ *equal roots so tgt*
- $\bullet^9$ $\text{Q} = (4, -3)$ **5 marks**

Notes

In (a)

1 $\bullet^4$ is only available if an attempt has been made to find the gradient from differentiation.

In (b)

2 $\bullet^6$ is only available for a numerical value of m.

3 An "= 0" must occur somewhere in the working between $\bullet^7$ and $\bullet^8$.

4 $\bullet^8$ is awarded for drawing a conclusion from the candidate's quadratic equation.

5 Candidates may substitute the equation of the parabola into the equation of the line. This is a perfectly acceptable approach.

Common Error 1

$\surd$	$\bullet^1$	$\frac{dy}{dx} =$
$\surd$	$\bullet^2$	$2x - 14$
X	$\bullet^3$	$2x - 14 = 0$ *so* $x = 7$ *so* $m = 7$
X	$\bullet^4$	$y - 5 = 7(x - 8)$
$X\surd$	$\bullet^5$	$y = 7x - 51$
$X\surd$	$\bullet^6$	$7x - 51 = -x^2 + 10x - 27$
$X\surd$	$\bullet^7$	$x^2 - 3x - 24 = 0$
$X\surd$	$\bullet^8$	$b^2 - 4ac = 105 \Rightarrow$ *line is not tgt*
X	$\bullet^9$	$--$

so award 6 marks

Alternative Marking 1 [Marks 8]

- $\bullet^8$ $b^2 - 4ac = 64 - 4 \times 16 = 0 \Rightarrow$ *line is a tangent*

Alternative Method 1 for (b)

- $\bullet^5$ $2x = y + 11$
- $\bullet^6$ $4y = -(y^2 + 22y + 121) + 20y + 220 - 108$
- $\bullet^7$ $y^2 + 6x + 9 = 0$
- $\bullet^8$ $(y+3)^2 = 0 \Rightarrow$ *equal roots so tgt*
- $\bullet^9$ $\text{Q} = (4, -3)$

Alternative Method 2 for (b)

- $\bullet^5$ *Find the equ. of the tgt to* 2*nd curve with grad.* 2 **stated or implied by •6**
- $\bullet^6$ $-2x + 10 = 2$
- $\bullet^7$ $\text{Q} = (4, -3)$
- $\bullet^8$ $y - (-3) = 2(x - 4)$
- $\bullet^9$ $y = 2x - 11$ *which is the same equ. as* (a) **stated explicitly**

4 The circles with equations $(x-3)^2+(y-4)^2=25$ and $x^2+y^2-kx-8y-2k=0$.
have the same centre. Determine the radius of the larger circle. **5**

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
4		5	C	G9	CN	06/55

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- $\bullet^1$ ic state centre of circle 1
- $\bullet^2$ ss equate x-coordinates, find k.
- $\bullet^3$ ic find radius of circle 1
- $\bullet^4$ ic substitute into the radius formula
- $\bullet^5$ ic process radius formula and compare.

Primary Method : Give 1 mark for each •

- $\bullet^1$ $C_1=(3,4)$
- $\bullet^2$ $k=6$
- $\bullet^3$ $R_1=5$
- $\bullet^4$ $R_2=\sqrt{(-3)^2+(-4)^2-(-12)}$ **or equivalent**
- $\bullet^5$ $\sqrt{37}>5$ *or* "2nd circle"

5 marks

Notes

1 $\bullet^2$ requires no justification.

2 Evidence for $\bullet^3$ may appear for the first time at the $\bullet^5$ stage.

3 If $R_1 = 5$ is clearly stated at the $\bullet^3$ stage, then it does not have to appear at the $\bullet^5$ stage for the conclusion to be drawn.

4 For any formula masquerading as the radius formula (e.g. see Common Error 2) , $\bullet^4$ and $\bullet^5$ are NOT available.

Alternative Method 1

- $\bullet^1$ $x^2+y^2-6x-8y+25=25$
- $\bullet^2$ $k=6$
- $\bullet^3$ $R_1=5$
- $\bullet^4$ $R_2=\sqrt{(-3)^2+(-4)^2-(-12)}$ **or equivalent**
- $\bullet^5$ $\sqrt{37}>5$ *or* "2nd circle"

Common Error 1

$\checkmark$	$\bullet^1$	$C_1=(3,4)$
$\checkmark$	$\bullet^2$	$k=6$
$\checkmark$	$\bullet^3$	$R_1=5$
X	$\bullet^4$	$R_2=\sqrt{(-3)^2+(-4)^2-12}$
$X\checkmark$	$\bullet^5$	$\sqrt{13}<5$ *or* "1st circle"

Common Error 2

$\checkmark$	$\bullet^1$	$C_1=(3,4)$
$\checkmark$	$\bullet^2$	$k=6$
$\checkmark$	$\bullet^3$	$R_1=5$
X	$\bullet^4$	$R_2=\sqrt{(-3)^2+(-4)^2+(12)^2}$
X	$\bullet^5$	$13>5$ *or* "2nd circle"

5 The curve $y = f(x)$ is such that $\frac{dy}{dx} = 4x - 6x^2$. The curve passes through the point $(-1, 9)$. Express y in terms of x. **4**

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
5		4	C/B	C18	CN	06/37

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- $\bullet^1$ ss know to integrate
- $\bullet^2$ pr integrate
- $\bullet^3$ ic substitute values
- $\bullet^4$ pr process constant

Primary Method : Give 1 mark for each ·

- $\bullet^1$ $y = \int \ldots$ **stated or implied by ·2**
- $\bullet^2$ $\frac{4}{2}x^2 - \frac{6}{3}x^3$
- $\bullet^3$ $9 = 2(-1)^2 - 2(-1)^3 + c$
- $\bullet^4$ $y = 2x^2 - 2x^3 + 5$ **stated explicitly**

4 marks

Notes

1 The equation "y = ………" must appear somewhere in the solution.

Common Error 1 Missing "equation"

$\surd$	$\bullet^1$	$y = \int ..$
$\surd$	$\bullet^2$	$\frac{4}{2}x^2 - \frac{6}{3}x^3$
$\surd$	$\bullet^3$	$9 = 2(-1)^2 - 2(-1)^3 + c$
X	$\bullet^4$	$c = 5$

award 3 marks

Common Error 2 : Not using (–1, 9)

$\surd$	$\bullet^1$	$y = \int ..$
$\surd$	$\bullet^2$	$\frac{4}{2}x^2 - \frac{6}{3}x^3$
X	$\bullet^3$	$2(-1)^2 - 2(-1)^3 + c = 0$
X	$\bullet^4$	$y = 2x^2 - 2x^3 - 4$

award 2 marks

Alternative Marking

- $\bullet^1$ $y = \int \ldots$
- $\bullet^2$ $\frac{4}{2}x^2 - \frac{6}{3}x^3$
- $\bullet^3$ $\left[\begin{array}{l} y = 2x^2 - 2x^3 + c \\ and \\ 9 = 2(-1)^2 - 2(-1)^3 + c \end{array}\right.$ **stated explicitly**
- $\bullet^4$ $c = 5$

6 P is the point (–1, 2, –1) and Q is (3, 2, –4).

(*a*) Write down $\overrightarrow{PQ}$ in component form. **1**

(*b*) Calculate the length of $\overrightarrow{PQ}$. **1**

(*c*) Find the components of a unit vector which is parallel to $\overrightarrow{PQ}$. **1**

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
6	a	1	C	G17	CN	06/59
	b	1	C	G16		
	c	1	B	G18		

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- •[1] ic state vector components
- •[2] pr find the length of a vector
- •[3] ic state unit vector

Primary Method : Give 1 mark for each •

•[1] $\overrightarrow{PQ} = \begin{pmatrix} 4 \\ 0 \\ -3 \end{pmatrix}$ **1 mark**

•[2] $|\overrightarrow{PQ}| = 5$ **1 mark**

•[3] $\begin{pmatrix} \frac{4}{5} \\ 0 \\ -\frac{3}{5} \end{pmatrix}$ **1 mark**

Note

In (a)

1 It is perfectly acceptable to write the components as a row vector eg $\overrightarrow{PQ} = \begin{pmatrix} 4 & 0 & -3 \end{pmatrix}$.

Treat $\overrightarrow{PQ} = (4, 0, -3)$ as bad form (i.e. not penalised).

In (b)

2 •[2] is not awarded for an unsimplified $\sqrt{25}$.

3 Beware of misappropriate use of the scalar product where, by coincidence, $\boldsymbol{p}.\boldsymbol{q} = 5$.

In (c)

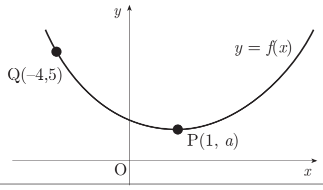
4 Accept $\frac{1}{5}\begin{pmatrix} 4 \\ 0 \\ -3 \end{pmatrix}$ for •[3].

7 The diagram shows the graph of a function $y = f(x)$.

Copy the diagram and on it sketch the graphs of

(a) $y = f(x-4)$ **2**

(b) $y = 2 + f(x-4)$ **2**



Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
7	a	2	C	A3	CN	06/new
	b	2	C	A3		

The primary method m/s is based on the following generic m/s. THIS GENERIC M/S MAY BE USED AS AN EQUIVALENCE GUIDE BUT ONLY WHERE A CANDIDATE DOES NOT USE THE PRIMARY METHOD OR ANY ALTERNATIVE METHOD SHOWN IN DETAIL IN THE MARKING SCHEME

- •¹ ic know translate parallel to x-axis, +ve dir.
- •² ic annotate points
- •³ ic know translate parallel to y-axis, +ve dir.
- •⁴ ic annotate points

Primary Method : Give 1 mark for each •

- •¹ translate 4 units right and annotate one point
- •² annotate the other point $[P'(5,a)\ Q'(0,5)]$

2 marks

- •³ translate (a) 2 units up and annotate one point
- •⁴ annotate the other point $[P''(5,a+2)\ Q''(0,7)]$

2 marks

Alternative Method

- •¹ translate 4 units right and annotate one point
- •² annotate the other point $[P'(5,a)\ Q'(0,5)]$
- •³ translate original $\begin{pmatrix}4\\2\end{pmatrix}$ and annotate one point
- •⁴ annotate the other point $[P''(5,a+2)\ Q''(0,7)]$

Notes

For (a)

1 A translation of $\begin{pmatrix}-4\\0\end{pmatrix}$ earns a maximum of 1 mark with both points clearly annotated and f(x) retaining its shape.

2 Any other translation gains no marks.

In the Primary method

For (b)

3 •³ and •⁴ are only available for applying the translation to the resultant graph from (a).

4 A translation of $\begin{pmatrix}0\\-2\end{pmatrix}$ earns a maximum of 1 mark with both points clearly annotated and the resultant graph from (a) retaining its shape.

5 Any other translation gains no marks.

In the Alternative method

For (b)

6 A translation of $\begin{pmatrix}4\\-2\end{pmatrix}, \begin{pmatrix}-4\\2\end{pmatrix}$ or $\begin{pmatrix}-4\\-2\end{pmatrix}$ applied to the original graph earns a maximum of 1 mark with both points clearly annotated and the resultant graph retaining its original shape.

7 Any other translation gains no marks.

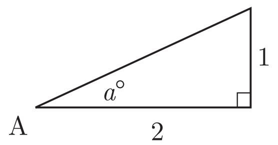
In either method

For (a) and (b)

8 For the annotated points, accept a superimposed grid or clearly labelled axes.

9 A candidate may choose to use two separate diagrams. This is acceptable.

8 The diagram shows a right-angled triangle with height 1 unit, base 2 units and an angle of $a°$ at A.



(*a*) Find the exact values of

(*i*) $\sin a°$

(*ii*) $\sin 2a°$. **4**

(*b*) By expressing $\sin 3a°$ as $\sin(2a+a)°$, find the exact value of $\sin 3a°$. **4**

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
8	a	4	C	T9	CN	06/44
	b	4	B	T8	CN	

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- $\bullet^1$ ic interpret diagram for sin(a°)
- $\bullet^2$ ss use double angle formula for sin(2A)
- $\bullet^3$ ic interpret diagram for $\cos(a°)$
- $\bullet^4$ pr substitute and complete
- $\bullet^5$ ss use compound angle formula
- $\bullet^6$ pr use double angle formula for cos(2A)
- $\bullet^7$ ic substitute
- $\bullet^8$ pr complete

Primary Method : Give 1 mark for each $\bullet$

- $\bullet^1$ $\sin(a°) = \dfrac{1}{\sqrt{5}}$
- $\bullet^2$ $\sin(2a°) = 2\sin(a°)\cos(a°)$
- $\bullet^3$ $\cos(a°) = \dfrac{2}{\sqrt{5}}$
- $\bullet^4$ $\sin(2a°) = \dfrac{4}{5}$ **4 marks**
- $\bullet^5$ $\sin(3a°) = \sin(2a°)\cos(a°) + \cos(2a°)\sin(a°)$
- $\bullet^6$ $\cos(2a°) = \dfrac{3}{5}$
- $\bullet^7$ $\sin(3a°) = \dfrac{4}{5}.\dfrac{2}{\sqrt{5}} + \dfrac{3}{5}.\dfrac{1}{\sqrt{5}}$
- $\bullet^8$ $\sin(3a°) = \dfrac{11}{5\sqrt{5}}$ **4 marks**

Note

1 Calculating approximate angles using arcsin and arccos gains no credit.

2 There are 3 processing marks $\bullet^4$, $\bullet^6$ and $\bullet^8$. None of these are available for an answer > 1.

3 sin(2a) = 0.8 and cos(2a) = 0.6 are the only two decimal fractions which may receive any credit.

4 Some candidates may double the height of the triangle and then call the base angle 2a. This error is equivalent to Common Error 1 illustrated on the right.

Common Error 1 An example of Incorrect formulae

$\checkmark$	$\bullet^1$	$\sin(a°) = \dfrac{1}{\sqrt{5}}$
X	$\bullet^2$	$\sin(2a°) = 2\sin(a°)$
X	$\bullet^4$	$\sin(2a°) = \dfrac{2}{\sqrt{5}}$
$\checkmark$	$\bullet^5$	$\sin(3a°) = \sin(2a°)\cos(a°) + \cos(2a°)\sin(a°)$
$\checkmark$	$\bullet^3$	$\cos(a°) = \dfrac{2}{\sqrt{5}}$
X	$\bullet^6$	$\cos(2a°) = \dfrac{4}{\sqrt{5}}$
$X\checkmark$	$\bullet^7$	$\sin(3a°) = \dfrac{2}{\sqrt{5}}.\dfrac{2}{\sqrt{5}} + \dfrac{4}{\sqrt{5}}.\dfrac{1}{\sqrt{5}}$
X	$\bullet^8$	$\sin(3a°) = \dfrac{8}{5}$

Common Error 2
An example based on a numerical error in Pythagoras

X	$\bullet^1$	$\sin(a°) = \dfrac{1}{\sqrt{3}}$
$\checkmark$	$\bullet^2$	$\sin(2a°) = 2\sin(a°)\cos(a°)$
$X\checkmark$	$\bullet^3$	$\cos(a°) = \dfrac{2}{\sqrt{3}}$
X	$\bullet^4$	$\sin(2a°) = \dfrac{4}{3}$
$\checkmark$	$\bullet^5$	$\sin(3a°) = \sin(2a°)\cos(a°) + \cos(2a°)\sin(a°)$
X	$\bullet^6$	$\cos(2a°) = 2\cos^2(a°) - 1 = \dfrac{5}{3}$ *or equivalent*
$X\checkmark$	$\bullet^7$	$\sin(3a°) = \dfrac{4}{3}.\dfrac{2}{\sqrt{3}} + \dfrac{5}{3}.\dfrac{1}{\sqrt{3}}$
X	$\bullet^8$	$\sin(3a°) = \dfrac{13}{3\sqrt{3}}$

9 $y = \dfrac{1}{x^3} - \cos 2x,\ x \neq 0,$ find $\dfrac{dy}{dx}$. 4

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
8		4	C/B	C3,C20	CN	06/79

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- $\bullet^1$ ss express in differentiable form
- $\bullet^2$ pr differentiate a term with a negative power
- $\bullet^3$ pr start to process a compound derivative
- $\bullet^3$ pr complete compound derivative

Primary Method : Give 1 mark for each •

- $\bullet^1$ x^{-3}
- $\bullet^2$ $-3x^{-4}$
- $\bullet^3$ $+\sin 2x$
- $\bullet^4$ $\times 2$

4 marks

Notes

1 For clearly integrating, correctly or otherwise, only $\bullet^1$ is available.

2 If you cannot decide whether a candidate has attempted to differentiate or integrate, assume they have attempted to differentiate.

10 A curve has equation $y = 7\sin x - 24\cos x$.

(a) Express $7\sin x - 24\cos x$ in the form $k\sin(x-a)$ where $k > 0$ and $0 \le a \le \frac{\pi}{2}$. **4**

(b) Hence find, in the interval $0 \le x \le \pi$, the x-coordinate of the point on the curve where the gradient is 1. **3**

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
10	a	4	C	T13	CR	06/97
	b	3	A/B	T17	CR	

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- •1 ss expand
- •2 ic compare coefficients
- •3 pr process k
- •4 pr process a
- •5 ic state result
- •6 ss set derivative = gradient
- •7 pr process 'x' from the derivative

Primary Method : Give 1 mark for each •

- •1 $k\sin(x)\cos(a) - k\cos(x)\sin(a)$ **stated explicitly**
- •2 $k\cos(a) = 7,\ k\sin(a) = 24$ **stated explicitly**
- •3 $k = 25$
- •4 $a = 1.29$

4 marks

- •5 $25\sin(x - 1.29)$
- •6 $\frac{dy}{dx} = 25\cos(x-1.29) = 1$
- •7 $x = 2.82$

3 marks

Notes

In (a)

1 $k\big(\sin(x)\cos(a) - \cos(x)\sin(a)\big)$ is acceptable for •1.

2 Treat $k\sin(x)\cos(a) - \cos(x)\sin(a)$ as bad form if •2 is gained.

3 No justification is required for •3.

4 •3 is not available for an unsimplified $\sqrt{625}$.

5 $25\big(\sin(x)\cos(a) - \cos(x)\sin(a)\big)$ is acceptable evidence for •1 and •3.

6 Candidates may use any form of the wave equation to start with as long as their final answer is in the form $k\sin(x-a)$. If it is not, then •4 is not available.

7 •4 is only available for
(i) an answer in radians which rounds to 1.3 OR
(ii) an answer given as a multiple of π *e.g.* $\frac{37}{90}\pi$.

8 $k\cos(a) = 7$ *and* $k\sin(a) = -24$ leading to a = 4.99 can only gain •4 if a comment intimating that this answer is not in the given interval is given.

In (b)

9 In (b) candidates have a choice of two starting points.
They can either start from $y = 25\sin(x-1.29)$ as shown in the Primary method OR
they can start from $\frac{dy}{dx} = 7\cos(x) + 24\sin(x)$. Either of these starting positions may be awarded •5.

10 Candidates who work in degrees will lose •6 for attempting to differentiate .

11 •7 is only available as a consequence of solving $\frac{dy}{dx} = 1$. Do not penalise "extra" solutions at the •7 stage (e.g. 6.04).

Common Error 1 Working in degrees

√	•1	$25\big(\sin(x)\cos(a) - \cos(x)\sin(a)\big)$
√	•2	$k\cos(a) = 7,\ k\sin(a) = 24$
√	•3	$k = 25$
X	•4	$a = 73.7$
√	•5	$25\sin(x - 73.7)$
X	•6	$\frac{dy}{dx} = 25\cos(x - 73.7) = 1$
√X	•7	$x = 161.4$

Award (a) 3 marks and (b) 2 marks

11 It is claimed that a wheel is made from wood which is over 1000 years old.
To test this claim, carbon dating is used.

The formula $A(t) = A_0 e^{-0.000124t}$ is used to determine the age of the wood, where A_0 is the amount of carbon in any living tree, $A(t)$ is the amount of carbon in the wood being dated and t is the age of the wood in years For the wheel it was found that $A(t)$ was 88% of the amount of carbon in a living tree.
Is the claim true? **5**

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
11		5	A/B	A30	CR	06/36

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- •[1] ic interpret information
- •[2] ic substitute
- •[3] ss take logarithms
- •[4] pr process
- •[5] ic interpret result

Primary Method : Give 1 mark for each •

- •[1] $A(t) = 0.88A_0$ **stated or implied by •[2]**
- •[2] $e^{-0.000124t} = 0.88$
- •[3] $\ln\left(e^{-0.000124t}\right) = \ln\left(0.88\right)$ **stated or implied by •[4]**
- •[4] $-0.000124t = \ln(0.88)$
- •[5] $t = 1031$ years so claim valid

5 marks

Notes

1 Candidates may choose a numerical value for A_0 at the start of their solution. Accept this situation.

2 •[5] is only available if •[4] has been awarded.

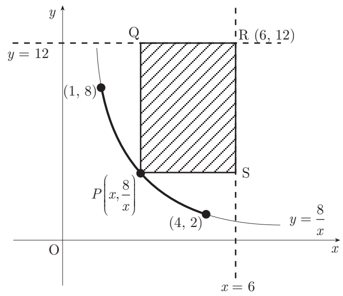
3 In following through from an error, •[5] is only available for a positive value of t.

Alternative Method 1 Graph and Calculator Solution

- •[1] $A(1000) = A_0 e^{-0.000124\times1000}$
- •[2] $0.883A_0$ *and* 1000 year old piece of wood contains 88.3% carbon.
- •[3] try a point where t > 1030
  e.g. $A(1050)$ getting $0.878A_0$
- •[4] sketch of y=$A_0 e^{-0.000124t}$ showing
  1. a monotonic decreasing function
  2. points representing eg (1000, 88.3%) etc
- •[5] observation that the point lies between the two plotted values for t and so claim valid.

12 PQRS is a rectangle formed according to the following conditions:

- it is bounded by the lines $x = 6$ and $y = 12$
- P lies on the curve with equation $y = \frac{8}{x}$ between (1, 8) and (4, 2)
- R is the point (6, 12).



(*a*) (i) Express the lengths of PS and RS in terms of x, the x-coordinate of P.

(ii) Hence show that the area, A square units, of PQRS is given by $A = 80 - 12x - \frac{48}{x}$. **3**

(*b*) Find the greatest and least possible values of A and the corresponding values of x for which they occur. **8**

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
12	a	3	A	C12	CN	06/20
	b	9	A/B	C12		

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- •1 ic interpret diagram to find PS
- •2 ic interpret diagram to find RS
- •3 ic complete proof
- •4 ic express in differentiable form
- •5 ss know to set derivative to zero
- •6 pr differentiate
- •7 pr process equation
- •8 pr evaluate area at the turning point
- •9 pr evaluate area at the end point
- •10 pr evaluate area at the end point
- •11 ic state conclusion

Primary Method : Give 1 mark for each •

- •1 $PS = 6 - x$
- •2 $RS = 12 - \frac{8}{x}$
- •3 $Area = \left(6 - x\right)\left(12 - \frac{8}{x}\right)$ and complete **3 marks**
- •4 $48x^{-1}$
- •5 $\frac{dA}{dx} = 0$
- •6 $-12 + 48x^{-2}$
- •7 $x = 2$
- •8 $A(2) = 32$
- •9 $A(1) = 20$
- •10 $A(4) = 20$
- •11 $\max.A = 32 \; at \; x = 2 \; \textbf{and}$ $\min.A = 20 \; at \; x = 1 \; or \; x = 4$ **8 marks**

Notes

1 For •3 there needs to be clear evidence that candidates have multiplied out the brackets in order to complete the proof.

2 An " = 0 " must appear somewhere in the working between •4 and •7.

3 At the •7 stage, ignore the omission or inclusion of x = –2.

4 •8 has to be as a consequence of solving $\frac{dA}{dx} = 0$.

5 •11 is only available if both end points have been considered.